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APPROXIMATE CALCULATION OF THE STATIC LONGITUDINAL STABILITY OF AIRPLANES

By Theodor Bienen

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By the static stability of an airplane is meant its ability to withstand forces and moments, which tend to disturb its state of equilibrium, by opposing greater forces and moments (whose production is the task of the tail group), and thus to retain its original equilibrium.

In considering static stability, no attention will be paid to the motions of the airplane, which are produced by the disturbing forces, as these come under the head of dynamic stability. Static longitudinal stability has been exhaustively treated in numerous articles, the first of which was published in Germany in "Flugsport," 1910, by H. Reissner.*

The accurate calculation of static stability, especially for multiplanes, is now very troublesome. This is especially noticeable, when such details as stagger and decalage (interinclination) are taken into account in calculating the mutual effect of the wings. It is a question as to whether such a de-

* "Eine einfache Methode zur angenäherten Berechnung der statischen Längsstabilität von Ein- und Doppeldeckern," from Zeitschrift für Flugtechnik und Motorluftschiffahrt, July 28, 1926, pp. 299-305. This treatise was taken from the course in aviation at the Aachen Technical High School, which explains its didactic character.
** See Fuchs-Hopf, "Aerodynamik," Berlin, 1922, p.310 ff., and the bibliography on p.459 of the same book.

tailed calculation is appropriate when other influences of fully as much importance, are more or less neglected. Such details include:

Effect of propeller slip stream on wings and tail group;

Deviations of the profile or wing section coefficients in experiments with models and full-size airplanes, resulting from different Reynolds Numbers and from differences always existing between the model and the full-size wing;

Differences between the assumed and actual lift distribution, not only between the upper and lower wings, but also throughout the span;

Differences between the computed and actual position of the center of gravity;

Inaccurate assumptions on the magnitude of the structural resistance and the point of application of its several coefficients, especially at different angles of attack.

It seems at least desirable to have some simple method for calculating quickly and with sufficient accuracy:

1. The correct position of the center of gravity;
2. The requisite tail-group dimensions;
3. The course of the wing and tail-group moments.

In our deductions, we will first replace the biplane (disregarding the effect of stagger, decalage and induced drag) by an equivalent monoplane, whose dimensions and position in space can be approximately determined in a simple manner.

In the computation of the balancing of the moments and in accord with the suggestion of Von Karman, at whose request we employed the following method, we adopted, as the point of reference for the moments, the intersection of the wing chord with the projection of the leading edge on the chord, i.e., the point to which the moment coefficients of experiments with models are generally referred. This resulted in certain simplifications, since we did not have to convert the lift and drag coefficients into normal force and tangential-force coefficients, in the well-known tedious manner, but could utilize directly the c_m values of the experiment with a model.

Replacement of the Biplane by a Monoplane

We start with a biplane (with stagger and decalage) according to Fig. 1 and seek an equivalent monoplane whose moment is

$$M = c_{mE}\ t_E\ F_{ges}\ q \qquad (1)$$

wherein the moment coefficient c_{mE} refers to the point of intersection of the wing chord of the desired monoplane with the projection of the leading edge on the chord. We therefore seek the momentary effective angles of attack x_o and x_u, y_o and y_u (Fig. 1) and the wing chord of an equivalent monoplane.

For an infinite aspect ratio, we can, in general, write

$$c_a = c_a'(\alpha)\ (\alpha + \delta),$$

where $c_a'(\alpha)$ is the derivative of c_a with respect to α,.

and $\alpha + \delta$ denotes the direction of the air stream relative to the $c_a = 0$ line (Fig. 1). For normal angles of attack, c_a' is a constant, to which we will subsequently return. Hence

$$c_{a\Sigma} F_{ges}\, q = (c_{ao} F_o + c_{au} F_u)\, q$$

$$c_a'(\alpha)\, \alpha = \frac{c_a'\,(\alpha + \delta - \sigma)\, F_o + c_a'\,(\alpha + \delta + \sigma)\, F_u}{F_{ges}}$$

from which the effective angle of attack of an equivalent monoplane (at first for an infinite aspect ratio) is found to be

$$\alpha = \alpha + \delta - \sigma\, \frac{F_o - F_u}{F_o + F_u} \qquad (2)$$

With a positive decalage of σ, the wing chord of an equivalent monoplane is inclined toward the mean chord (Mittellage) at an angle of $\sigma \frac{F_o - F_u}{F_{ges}}$. The angle of attack $\alpha + \delta$ is then diminished by this amount, but, with a negative decalage, it is increased (provided $F_o > F_u$).

The conversion of the measurement to the correct aspect ratio is then made in the usual manner. With the most favorable lift distribution, we obtain the angle of attack

$$\overline{\alpha} = \alpha_M + \Delta\, \alpha$$

$$\Delta\, \alpha = \frac{c_a}{\pi} \left\{ \kappa\, \frac{\Sigma\, F}{b_1^{\,2}} - \left(\frac{F}{b^2}\right)_M \right\}$$

The index M denotes the values for the model measurement. Hence $\left(\frac{F}{b^2}\right)_M$ denotes the aspect ratio of the model; $\Sigma\, F$, the

total wing area of the biplane; b_1, the span; while κ is a function of the ratio, gap to span, and of the upper span to the lower span, and of the lift distribution.*

Thus we ultimately obtain

$$c_a = f\left(\bar{\alpha} - \sigma\,\frac{F_o - F_u}{F_{ges}}\right)$$

Likewise we can put

$$c_{mE} = f\left(\bar{\alpha} - \sigma\,\frac{F_o - F_u}{F_{ges}}\right)$$

The quantity $\sigma\,\dfrac{F_o - F_u}{F_{ges}}$, which represents the effect of the decalage, is generally negligible. The other quantities, by disregarding the decalage, are reduced very simply to

$$\left.\begin{aligned} t_E &= \frac{t_o\,F_o + t_u\,F_u}{F_{ges}} \\ x_o &= x\,\frac{F_u}{F_{ges}} \\ y_o &= y\,\frac{F_u}{F_{ges}} \end{aligned}\right\} \qquad (3)$$

If, for comparison, we compute the moment of a biplane and of an equivalent monoplane, both referred to the projection of the leading edge of the monoplane on the wing chord, we obtain,

* Prandtl, "Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," Part II, 1923, p.15 ff and p.37 ff. The bending effect, which is conspicuous with a relatively small gap, can be disregarded in this approximate computation. In like manner, we can determine the change in the induced drag according to the well-known formula

$$\Delta c_w = \frac{c_a^2}{\pi}\left\{\kappa\,\frac{\Sigma F}{b_1^2} \quad \left(\frac{F}{b^2}\right)_M\right\}$$

and therefrom

$$\bar{c}_w = c_{wM} + \Delta c_w$$

by omitting unimportant members,

$$c_{mo}\ t_o\ F_o + c_{mu}\ t_u\ F_u - c_{ao}\ \cos\alpha\ \ F_o\ x\ \frac{F_u}{F_{ges}}$$

$$+ c_{au}\ \cos\alpha\ \ F_u\ \frac{x\ F_o}{F_{ges}}$$

$$= c_{mE}\ (t_o\ F_o + t_u\ F_u).$$

We can now write

$$c_m = c_m'\ (c_a)\ c_a + c_{mp},$$

$$c_a = c_a'(\alpha)\ \ (\alpha + \delta).$$

By substituting the corresponding values right and left, we obtain

$$c_m'\ c_a'\ [(\alpha + \delta - \sigma)\ t_o\ F_o + (\alpha + \delta + \sigma) t_u\ F_u] +$$

$$+ c_{mp}\ [t_o\ F_o + t_u\ F_u] + c_a'\ \cos\alpha\ \frac{F_o\ F_u\ x}{F_{ges}}\ 2\,\sigma$$

$$= [c_m'\ c_a'\left(\alpha + \delta - \sigma\ \frac{F_o - F_u}{F_{ges}}\right) + c_{mp}]$$

$$(t_o\ F_o + t_u\ F_u).$$

It is obvious that, for $\sigma = 0$, the right and left sides agree. If $\sigma \neq 0$, we would then have to add to the right side (disregarding the higher order of magnitude of the small members)

$$2\ c_a'\ \sigma\ x\ \frac{F_o\ F_u}{F_{ges}}\ \cos\alpha,$$

so that the expression for the moment would assume the following form:

$$M = t_E\ F_E\ q\left(c_{mE} + 2\ c_a'\ \sigma\ \frac{x}{t_E}\ \ \frac{F_o\ F_u}{F_E^{\,2}}\ \cos\alpha\right)$$

The second term in the brackets, however, can obviously be omitted, if both the stagger and the decalage do not simultaneously have large values. This term is positive when both stagger and decalage are simultaneously either positive or negative and it then increases the moment. When the signs are unlike, however, the expression is negative.

Calculation of the Static Stability

We will now turn to the calculation of the static stability of a monoplane, to which, as we demonstrated in the preceding section, the biplane can be reduced.

We will first consider the stability in horizontal flight. Fig. 2 represents the airplane under consideration. The origin of the coordinates is located at the reference point of the moments and all quantities are calculated as positive downward and backward. The moments are balanced $(\alpha = 0)$ at the angle of attack i. The angle α refers to the mean-chord line (Mittellinie) and therefore has a different meaning from that given it above, where α is the angle of attack of the wing chord.

S = propeller thrust turning about the center of moments on the lever arm s. (S is assumed to be parallel to the mean-chord line);

G = total weight of airplane;

r = distance back to center of gravity;

h = distance down to center of gravity;

W = total structural drag acting on lever arm w ($W \times w$ is assumed to be independent of α);

l = distance of center of lift of tail group from the center of moments;

f = total area of horizontal tail planes.

Then the total moment of the airplane, referred to the normal center of moments of the wing section, is

$$M = q\,F\,t\left\{c_m - c_{w\,ges}\,\frac{s}{t}\,\frac{1}{\cos\alpha} - c_a\,\frac{r\cos\alpha - h\sin\alpha}{t}\right.$$

$$\left. + c_{ws}\,\frac{w}{t} + \frac{l}{t}\,\frac{f}{F}\,c_{nl}\left(\cos\alpha - \frac{l_a}{l}\sin\alpha\right)\right\} \qquad (4)$$

In this formula, c_m = the contribution of the wing,

$c_{w\,ges}\,\frac{s}{t}\,\frac{1}{\cos\alpha}$ = the contribution of the propeller thrust (in unaccelerated flight),

$c_a\,\frac{r\cos\alpha - h\sin\alpha}{t}$ = the contribution of the total weight, applied at the center of gravity of the airplane,

$c_{ws}\,\frac{W}{t}$ = the contribution of the structural drag, which, as already mentioned, is assumed to be independent of the angle of attack,

$\frac{l}{t}\,\frac{f}{F}\,c_{nl}\left(\cos\alpha - \frac{l_a}{l}\sin\alpha\right)$ = the contribution of the horizontal tail plane ($c_{nl}\,f$ = normal force coefficient times the area of the horizontal tail plane).

We now transform the expression for the tail plane and com-

pute the angle of attack i' (Fig. 2) in such manner that the tail moment vanishes for $\alpha = 0$ (hence for the balanced flight condition).

At first we can disregard $\frac{l_a}{l} \sin\alpha$ in comparison with $\cos\alpha$. Furthermore, approximately

$$c_{n l} = c_{a l} \cos\alpha = c_a{}'_{(\alpha)} \ (\alpha + i' + \Delta\,\alpha_F) \cos\alpha .$$

$c_a{}'$ is a constant for the existing angle-of-attack region. With sufficient accuracy, as demonstrated by the experimental results, we can write

$$c_a{}'\,(\alpha) = \frac{5.25}{1 + 1.67\,\frac{F}{b^2}} \tag{5}$$

$\Delta\,\alpha_F$ is the downwash angle produced, by the downwash from the wings, at the position of the horizontal tail plane. It is

$$\begin{aligned} \Delta\,\alpha_F &\cong - \frac{2}{\pi} \left(\frac{F}{b^2}\right)_F c_a{}^* \\ &= - \frac{2}{\pi} \left(\frac{F}{b^2}\right)_F c_a{}'_F \ (\alpha + \delta). \end{aligned}$$

The index F shows that the aspect ratio of the wing is meant.

If the airplane is to be balanced for δ (Fig. 2), with a

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* This downwash angle was produced by elliptical lift distribution resulting from the descending vortex, while the still less important effect of the supporting vortex and also the effect of the distortion (rolling up) of the vortex band, which occurred at large c_a values, was disregarded. See Fuchs-Hopf "Aerodynamik," Berlin, 1922, p.145, and especially, H. B. Helmbold, "Ueber die Berechnung des Abwindes hinter einem rechteckigen Flügel," Z.F.M. 1925, p.291. Our formula, aside from the disregard of the supporting vortex, represents an upper limit for the downwash angle. It is sufficiently accurate for practical purposes.

symmetrical cross section of the horizontal tail plane, we accordingly have

$$i' = \frac{2}{\pi}\left(\frac{F}{b^2}\right)_F c_a{}'_F\ \delta \tag{6}$$

After inserting this, we have

$$c_{n\,l} = c_a{}'_l\ \alpha \cos\alpha\ \left[1 - \frac{2}{\pi}\left(\frac{F}{b^2}\right)_F c_a{}'_F\right],$$

$$= c_a{}'_l\ \alpha \cos\alpha\ (1 - \bar{c}_a{}'_F) \tag{7}$$

Herein

$$c_a{}'_l = \frac{5.25}{1 + 1.67\left(\frac{f}{b^2}\right)_l}\ ;\ \bar{c}_a{}'_F = \frac{2}{\pi}\left(\frac{F}{b^2}\right)_F \frac{5.25}{1 + 1.67\left(\frac{F}{b^2}\right)_F} \tag{5a}$$

The indexes l and F refer respectively, to the tail and wing. We accordingly have

$$\frac{M}{q\ F\ t} = c_m - c_w\ \text{ges}\ \frac{s}{t}\ \frac{1}{\cos\alpha} - c_a\ \frac{r\cos\alpha - h\sin\alpha}{t} + c_{ws}\frac{w}{t}$$

$$+ \frac{l}{t}\,\frac{f}{F}\ c_a{}'_l\ \alpha\ (1 - \bar{c}_a{}'_F)\ \cos^2\alpha \tag{4a}$$

Taken in order, the terms on the right side indicate:

The moment coefficient of the wing,

" " " " " propeller thrust,

" " " produced by the weight of the airplane,

" " " of the structural or parasite drag,

" " " " " tail.

The moment coefficients of the propeller thrust and structural drag are generally of no importance.

The airplane is to be balanced for the angle of incidence

i (Fig. 2), hence for $\alpha = 0$. For this case we have

$$\frac{M}{q\ F\ t} = c_m - c_w\ ges\ \frac{s}{t} - c_a\ \frac{r}{t} + c_{ws}\ \frac{w}{t} = 0.$$

From this we obtain the expression for the requisite distance aft of the center of gravity

$$\frac{r}{t} = \frac{c_{mo} - c_w\ ges\ o\ \frac{s}{t} + c_{ws}\ \frac{w}{t}}{c_{ao}}, \tag{8}$$

wherein the values c_{mo}, c_w ges o, and c_{ao} are to be inserted for the corresponding angle of incidence i ($\alpha = 0$).

All the quantities are now known for calculating the moments, excepting l f for the tail moment. In order to find this, we add the other moment coefficients and plot them against a or c_a. Then l f is so determined that, according to the degree of stability desired, the moment coefficient of the horizontal tail plane for the existing a or c_a is equal to or greater than the maximum wing or other moment.

Attention is hereby called to the fact that s and h are calculated positively downward from the reference point of the moments (Fig. 2) and are therefore partly negative on biplanes and low-wing monoplanes. It is known that the small angle of attack combined with large drag coefficients in some airfoils (e.g., those with leading edges sharply curved downward) can be dangerous for low-winged monoplanes, is there is not a sufficiently large stabilizer provided.

Our formula needs to be supplemented for steep gliding flight with engine stopped. In this case, the weight of the airplane can be divided into one component in the direction of the lift and another in the direction of the drag (Fig. 3). In unaccelerated flight, these components are respectively equal to the lift and drag, but in opposite directions. In equation (4a) for the total moment, the expression

$$c_{w\ ges}\ \frac{s}{t}\ \frac{1}{\cos\alpha}$$

is then to be replaced by

$$c_{w\ ges}\ \frac{r\sin\alpha + h\cos\alpha}{t}\ ;$$

we must therefore find whether

$$\frac{s}{t\ \cos\alpha} \gtrless \frac{r}{t}\sin\alpha + \frac{h}{t}\cos\alpha .$$

The moment formula would then read:

$$\frac{M}{q\ F\ t} = c_m - c_{w\ ges}\ \frac{r\sin\alpha + h\cos\alpha}{t} - c_a\ \frac{r\cos\alpha - h\sin\alpha}{t}$$

$$+ c_{ws}\ \frac{W}{t} + \frac{l}{t}\ \frac{f}{F}\ c_a{}'_l\ \alpha\ (1 - \overline{c}_a{}'_F)\ \cos^2\alpha \qquad (4b)$$

This is of no practical importance, since the moment coefficient produced by the drag is generally small in comparison with the other quantities and, besides, $s/\cos\alpha$ can hardly be smaller than $r\sin\alpha + h\cos\alpha$. Moreover, the last equation (4b) also applies to engineless airplanes (gliders).

The expression for the requisite aft position on a glider

can be found by putting, as before, $\alpha = 0$. We then have

$$\frac{r}{t} = \frac{c_{mo} - c_{w\ ges\ o}\frac{h}{t} + c_{ws}\frac{w}{t}}{c_{ao}} \tag{8a}$$

In closing, we wish to deduce one more simple formula for the approximate calculation of the dimensions of the horizontal tail plane. For this purpose, we write the equation for the equilibration of the moments in the following form:

$$\frac{M}{q\ F\ t} = c_m'(c_a)\ c_a'(\alpha)\ (\alpha + \delta) + c_{mp} - \left(\frac{c_a^2\ F}{\pi\ b^2} + c_{ws}\right)\frac{s}{t} -$$

$$- c_a'(\alpha)\ (\alpha + \delta)\left(\frac{r}{t} + \alpha\ \frac{h}{t}\right) + \frac{l}{t}\ \frac{f}{F}\ c_a'{}_l\ \alpha\ (1 - \bar{c}_a'{}_F), \tag{9}$$

in which we develop $\sin\alpha$ and $\cos\alpha$ and disregard the terms with higher powers of α.

The stability formula now reads

$$\frac{\partial M}{\partial \alpha} > 0 \tag{10}$$

or, expressed in words, if we are to have stability, then with increasing angle α, a more rapidly increasing positive (hence nose-heavy) moment must be produced and, conversely, with a negatively increasing angle α, a greater falling negative (hence, tail-heavy) moment must be produced. Or, more briefly, a restoring moment must be produced in disturbances of the state of equilibrium.

We differentiate equation (9) with respect to α and obtain

$$\frac{\partial}{\partial \alpha} \frac{M}{q\,F\,t} = c_a{}' \left[c_m{}' - \frac{r}{t} - \frac{2\,c_a\,F}{\pi\,b^2}\,\frac{s}{t} + \frac{h}{t}\,(2\,\alpha + \delta) \right]$$

$$+ \frac{l f}{t F}\, c_a{}'_l\, (1 - \overline{c}_a{}'_F) \gtreqless 0.$$

If we now disregard the terms containing s/t and h/t, and write approximately for r/t the expression c_{mo}/c_{ao} - equation (8), we finally obtain

$$\frac{l}{t}\;\frac{f}{F} \geqq \frac{c_a{}'_F \left(\frac{c_{mo}}{c_{ao}} - c_m{}'(c_a) \right)}{c_a{}'_l\,(1 - c_a{}'_F)} \tag{11}$$

An Example

On account of our many omissions, we considered it advisable to test, by an example, the resulting differences, as compared with the accurate method of computation. For this purpose, we chose the example published by the D.V.L. (Deutsche Versuchsanstalt für Luftfahrt). The dimensions of the biplane considered are represented in Fig. 4, and are numerically as follows:

	Upper Wing	Lower Wing
Area	F_o=19.2 m² (206.7 sq.ft.)	F_u=18.0 m² (193.8 sq.ft.)
Span	b_o=12.0 m (39.37 ft.)	b_u=12.0 m (39.37 ft.)
Mean chord	t_o= 1.6 m (5.25 ft.)	t_u= 1.5 m (4.92 ft.)
Airfoil	Göttingen 398	Göttingen 398
Dist. aft	r_o= 0.82 m(2.69 ft.)	r_u= 0.24 m(0.79 ft.)
Dist. down	h_o= 0.93 m(3.05 ft.)	h_u= 0.67 m(2.20 ft.)

Decalage	$\sigma = 1^\circ$
Stagger	$\beta = 20^\circ$
Gap	$h = 1.6$ m (5.25 ft.)
Elevator:	
Area	$f = 3.8$ m² (40.9 sq.ft.)
Span	$b_H = 3.74$ m (12.27 ft.)
Chord	$t_H = 0.95$ m (3.12 ft.)
Distance of center of gravity from elevator axis,	$l = 4.44$ m (14.57 ft.)
Angle of incidence of elevator to mean chord of wings,	$\sigma = 0.4^\circ$
Coef. of structural drag,	$c_{ws} = 0.02$
Lever arm of structural drag,	0.15 m (0.49 ft.)
Height of propeller axis above center of gravity,	0.2 m (0.66 ft.)

The wings are balanced at an angle of attack of 4.5° and $c_a = 0.77$. If the position of the equivalent monoplane is calculated according to the above formulas, the distances from the leading edge to the center of gravity are

Aft $r = 0.553$ m (1.81 ft.)

Down $h = 0.150$ m (0.49 ft.)

According to equation (8), a distance aft of $r = 0.558$ m (1.83 ft.) would be required. The angle of incidence of the elevator to the mean chord of the wings is found according to equation (6) to be $\sigma = 0.6^\circ$ (against 0.4°).

In Fig. 5, the airplane polar is plotted accurately and the

points are also given (+) for the approximate polar, disregarding the stagger and decalage. (The "bending effect" is also disregarded here.)

Fig. 6 shows the course of the moment curve both by the accurate method and by the approximate method. The moment curves naturally differ, for one refers to the leading edge of the equivalent monoplane and the other to the center of gravity of the airplane. In both cases, the wing moment is zero, with almost the same angle of attack.

On the basis of the preceding considerations and of the example, I think it is not too much to claim that the simple method shown is entirely satisfactory for practical purposes. The deviations in comparison with the accurate method of computation fall within the limits of the errors in computing with a slide rule.

Summary

A decalaged and staggered biplane can be approximately replaced by a monoplane whose aspect ratio is given by $\kappa \frac{\Sigma F}{b_1^2}$. The moment of this monoplane is

$$M = c_{mE}\ t_E\ F_{ges}\ q \qquad (1)$$

in which c_{mE} is the moment coefficient of the given airfoil calculated for the above aspect ratio,

$$t_E = \frac{t_o\ F_o + t_u\ F_u}{F_{ges}} \qquad (3)$$

The airfoil, with positive stagger σ toward the mean chord (Fig. 1), is negatively inclined with the angle

$$\sigma \frac{F_o - F_u}{F_{ges}} , \tag{2}$$

while the aft and depth position of the equivalent monoplane with the notations of Fig. 1 is given by the expressions

$$\left.\begin{aligned} x_o &= x \frac{F_u}{F_{ges}} \\ y_o &= y \frac{F_u}{F_{ges}} \end{aligned}\right\} \tag{3}$$

The course of the total moment for an airplane is given by the equation

$$\frac{M}{q\ F\ t} = c_m - c_w\ ges \frac{s}{t\ \cos\alpha} - c_a \frac{r \cos\alpha - h \sin\alpha}{t} + c_{ws} \frac{w}{t}$$

$$+ \frac{l}{t}\ \frac{f}{F}\ c_a{'}_l\ \alpha\ (1 - \bar{c}_a{'}_F) \cos^2\alpha , \tag{4b}$$

the last term on the right being the coefficient of the tail moment, in which

$$c_a{'}_l = \frac{5.25}{1 + 1.67 \left(\frac{f}{b^2}\right)_l} \tag{5}$$

$$\bar{c}_a{'}_F = \frac{2}{\pi} \left(\frac{F}{b^2}\right)_F \frac{5.25}{1 + 1.67 \left(\frac{F}{b^2}\right)_F} \tag{5a}$$

The aft position of the center of gravity is given by the equations

$$\frac{r}{t} = \frac{c_{mo} - c_w \text{ ges } o \frac{s}{t} + c_{ws} \frac{w}{t}}{c_{ao}} \quad \text{(Engine driven airplane)} \qquad (8)$$

and

$$\frac{r}{t} = \frac{c_{mo} - c_w \text{ ges } \frac{h}{t} + c_{ws} \frac{w}{t}}{c_{ao}} \quad \text{(Engineless airplane)} \qquad (8a)$$

From the stability condition, we obtain, for the size of the elevator, approximately

$$\frac{lf}{tF} = \frac{c_a'_F \left(\frac{c_{mo}}{c_{ao}} - c_m'(c_a)\right)}{c_a'_l \, (1 - \bar{c}_a'_F)} \qquad (11)$$

S u p p l e m e n t

Professor Von Karman called my attention to the following method by which allowance can be made for the effect of the stagger on the stability.

Since the effect is only slight, we will be satisfied with an approximate calculation, assuming, as the basis of our deductions, that the upper and lower wings do not differ very greatly from one another, either in their dimensions or in the conditions of flow to which they are subjected.

We will consider the effect of the lower supporting vortex on the upper. The circulation of the lower vortex follows from the expression

$$\frac{\gamma}{g} \Gamma_u \, v \, b_u = \gamma \frac{v^2}{2g} c_{au} F_u$$

to

$$\Gamma_u = \frac{c_{au}}{2} v \frac{F_u}{b_u} .$$

A vortex of the strength Γ generates, at the distance r, a velocity of the magnitude

$$\Delta v = \frac{\Gamma}{2 \pi r},$$

perpendicular to the radius vector.

In our case, the following speed increment is added for the upper wing, as the result of the circulation about the lower wing.

$$\Delta v_{ou} = \frac{\Gamma_u}{2 \pi r} = \frac{c_{au}}{4 \pi} \frac{F_u}{b_u} \frac{v}{r}.$$

As a result of this speed increment, the upper wing, with a positive stagger of β degrees (Fig. 7), suffers a change in the angle of attack of approximately

$$\Delta \alpha_{ou} = \frac{\Delta v_{ou} \sin (\beta - \alpha)}{v}.$$

If we disregard the speed increment, we obtain, as a result of the change in the angle of attack, the following lift increment for the upper wing:

$$\Delta A_{ou} = \Delta \alpha_{ou} c_a' \gamma \frac{v^2}{2g} F_o \sin (\beta - \alpha)$$

$$= \frac{c_{au}}{4 \pi} \frac{F_u}{b_u} \frac{1}{r} c_a' \gamma \frac{v^2}{2\gamma} F_o \sin (\beta - \alpha).$$

For the lower wing we obtain a reduction of

$$\Delta A_{uo} = \frac{c_{ao}}{4 \pi} \frac{F_o}{b_o} \frac{1}{r} c_a' \gamma \frac{v^2}{2g} F_u \sin (\beta - \alpha).$$

in the lift.

Under the initial suppositions, we can now assume that the total lift is not changed and that

$$\Delta A_{ou} = - \Delta A_{uo}$$

or that

$$\Delta A = \frac{c_a}{4\pi} \; \frac{t}{r} \; c_a' \; \gamma \; \frac{v^2}{2\,g} \; F \sin(\beta - \alpha);$$

in which t denotes the chord and F the wing area.

Due to the change in the lift, there is generated, however, at positive angles of attack, a tail-heavy moment of

$$\Delta M = - \Delta A \; r \sin(\beta - \alpha)$$
$$= - \frac{c_a}{4\pi} \; t \; c_a' \; \gamma \; \frac{v^2}{2\,g} \; F \sin^2(\beta - \alpha),$$

the corresponding moment coefficient being

$$\Delta c_m = \frac{\Delta M}{\gamma \; \frac{v^2}{2\,g} \; t_E \; F_{ges}} = - \frac{c_a}{8\,\pi} \; c_a' \; \sin^2(\beta - \alpha).$$

The angle $\beta - \alpha$ may be designated as the effective angle of stagger. When the angle of attack is equal to the angle of stagger, the effect of the stagger vanishes. It also vanishes when the lift becomes zero. The coefficient reaches its maximum value at

$$\alpha \cong \frac{1}{3} (\beta - 2\,\delta),$$

in which δ is again the angle between the line $c_a = 0$ and the mean-chord line (Fig. 7).

In Fig. 8, the course of the supplementary moment coeffici-

ent is plotted against the angle of attack $\alpha + \delta$ for the Göttingen airfoil 426, with staggers of 0°, $+20^\circ$, -20°.

The effect is generally very slight. The tail-heavy moment coefficients, occurring with negative stagger (which is very seldom met with) at large angles of attack, are small in comparison with those of a single wing (monoplane).

The following are the necessary formulas for the stability calculation, given in the more accurate form. First we have for the wing moment of the biplane

$$M = \gamma \frac{v^2}{2g} t_E F_{ges} \left[c_{mE} + 2\, c_a{}' \sigma \frac{x}{t_E} \frac{F_o F_u}{F_{ges}{}^2} \cos \overline{\alpha} - \frac{c_a\, c_a{}'}{8\pi} \sin^2 (\beta - \alpha) \right] \tag{12}$$

The second term in the brackets comes from the stagger and decalage. Since we can put $\cos \overline{\alpha}$ as approximately 1, this term produces only a parallel shifting of the c_m values. The third term is simply a result of the stagger.

In our moment formula (4a), we would therefore have to substitute the bracketed expression from equation (12) in place of c_m.

For the more accurate location of the center of gravity, we write

$$\Delta c_m = \frac{c_a{}'^2}{8\pi} (\alpha + \delta) \sin^2 (\beta - \alpha),$$

in which α denotes the angle between the wind direction and the

mean chord (Figs. 2 and 7); δ, the angle between the mean chord and the line $c_a = 0$; β, the angle of stagger (with reference to a perpendicular to the mean chord). Corresponding to equation (8), we then have, for the location of the center of gravity,

$$\frac{r}{t} = \frac{c_{mo} + 2\ c_a{}'\ \sigma\ \frac{x}{t_E}\ \frac{F_o\ F_u}{F_{ges}{}^2} - \frac{c_a{}'^2}{8\ \pi}\ \delta\ \sin^2\beta - c_{w\ ges}\ \frac{s}{t} + c_{ws}\ \frac{s}{t}}{c_{ao}} \tag{13}$$

Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.

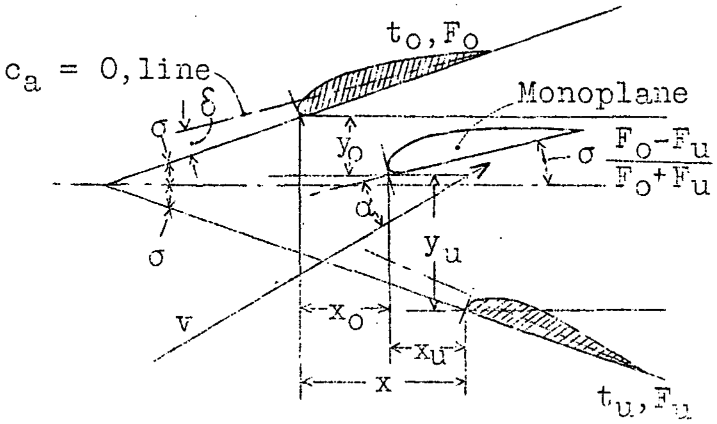

Fig.1 Biplane replaced by equivalent monoplane

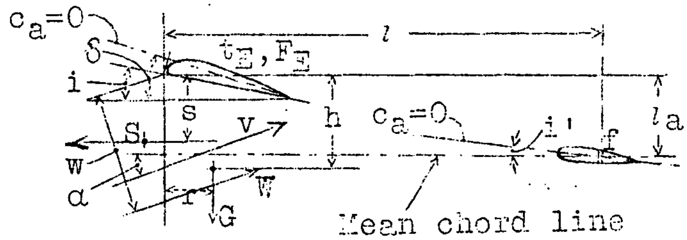

Fig.2 Airplane measurements and quantities required for computation.

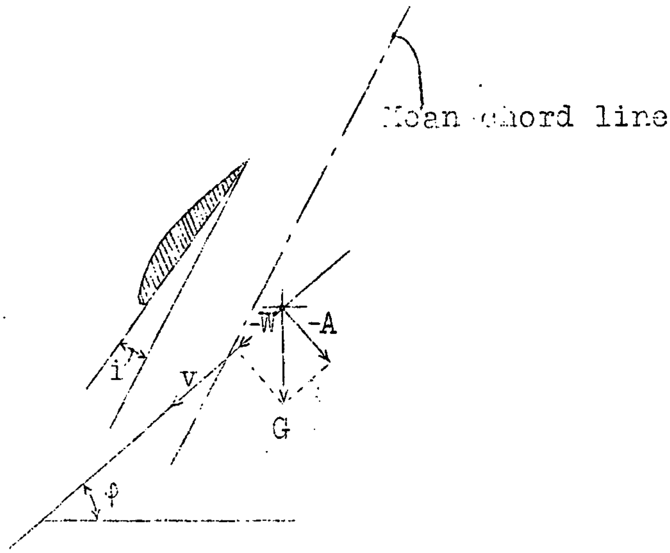

Fig.3 Airplane in steep glide.

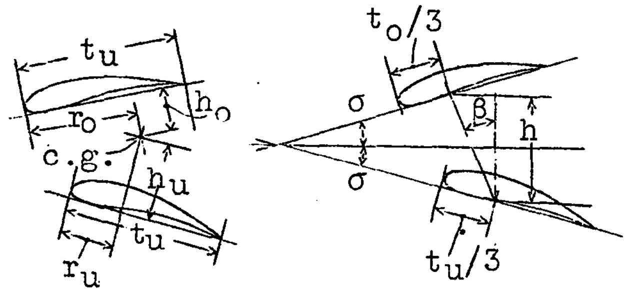

Fig.4 Diagram of wing dimensions.

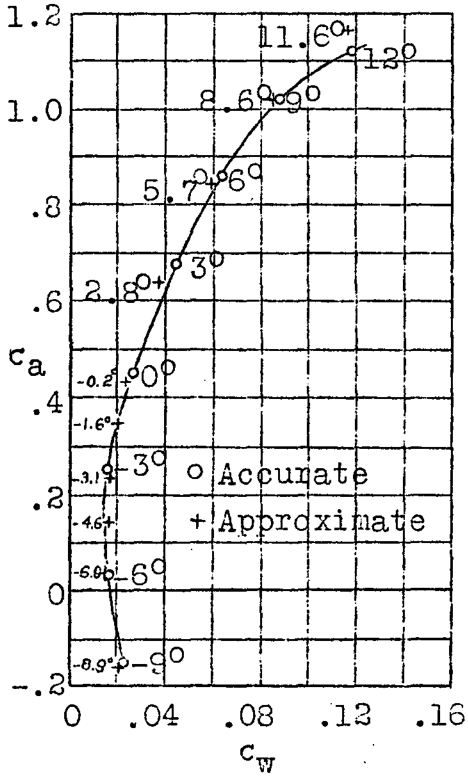

Fig.5 Course of biplane polar by both accurate and approximate computation.

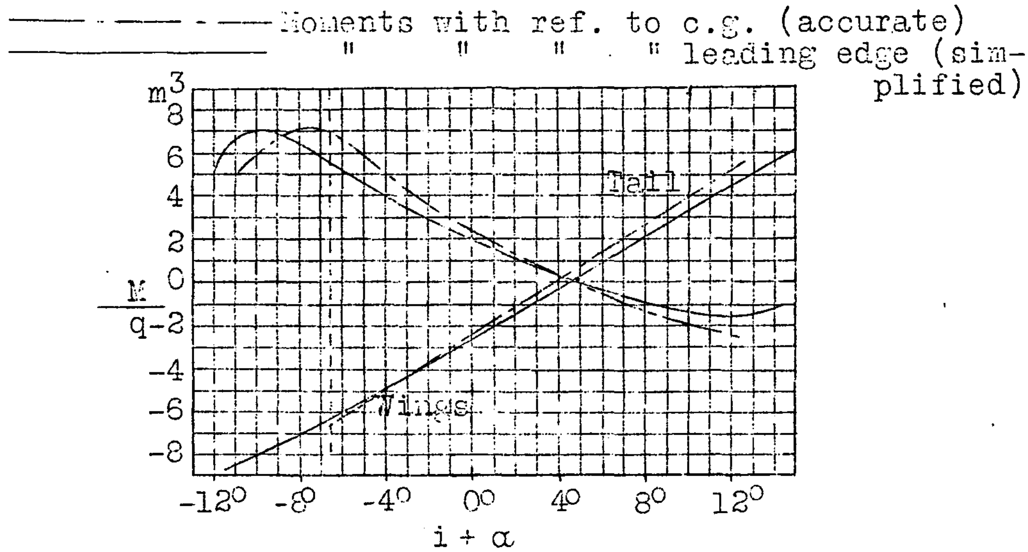

Fig.6 Moment curves.

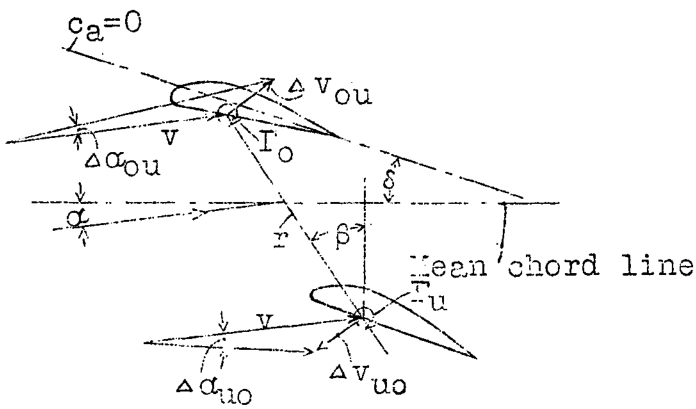

Fig.7 Effect of stagger on the effective angle of attack.

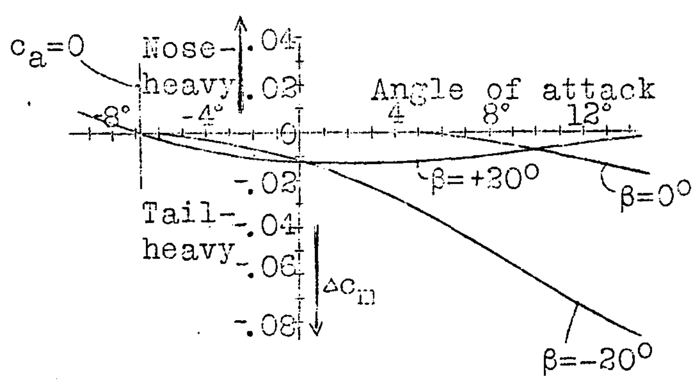

Fig.8 Course of supplemantary moment coefficient produced by various angles of stagger.